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TECHNICAL NOTE

Comprehensive correlations for laminar mixed convection line plume and wall plume

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1. INTRODUCTION

THE MIXED convection line plume and wall plume arising from a horizontal line source of heat or mass in a uniform vertical flow have been studied extensively [1–8]. However, the numerical results are available only for several specific Prandtl numbers. For convenience of engineering applications, correlation equations valid over the entire buoyancy regime for any Prandtl number are needed. The only correlation of the centerline temperature of a mixed convection line plume has been developed by Haaland and Sparrow [4]. They have also presented an exact solution of the centerline temperature for the forced convection case, which is applicable to any Prandtl number. The closed-form solution and correlation of a pure free convection line plume are not available. For a wall plume along an adiabatic plane surface, no correlation of the surface temperature for the forced, the free, or the mixed convection case has been reported.

In this paper, we have developed a simple and very accurate correlation equation for predicting the centerline temperature of a mixed convection line plume and the surface temperature of a mixed convection wall plume over the entire range of buoyancy for any Prandtl number. The correlation is in terms of an appropriate mixed convection parameter and the centerline temperature or the surface temperature of the limiting cases of pure forced convection plumes and pure free convection plumes for each specific Prandtl number. Very accurate correlations of the forced convection plumes and the free convection plumes for any Prandtl number between 0.001 and infinity have also been proposed.

2. CORRELATION OF MIXED CONVECTION PLUMES

The correlation equations for predicting the centerline temperature of a mixed convection line plume and the surface temperature of a mixed convection wall plume can be developed by using the properly defined [8] mixed convection parameter

$$\xi = [1+(\omega Re)^{1/2}/(\sigma Ra)^{1/5}]^{-1} \tag{1}$$

and dimensionless temperature

$$\begin{aligned}\theta(\xi,\eta) &= [(T-T_\infty)/T^*][(\omega Re)^{1/2}+(\sigma Ra)^{1/5}] \\ &= [(T-T_\infty)/T^*](\omega Re)^{1/2}/(1-\xi) \\ &= [(T-T_\infty)/T^*](\sigma Ra)^{1/5}/\xi \end{aligned} \tag{2}$$

where the plume characteristic temperature $T^* = Q/\rho c_p \alpha$, the local Reynolds number $Re = u_\infty x/\nu$, the local Rayleigh number $Ra = g\beta T^* x^3/\alpha\nu$, and

$$\omega = Pr/(1+Pr)^{1/3} \tag{3}$$

$$\sigma = Pr/(1+Pr)^{1/2} \quad \text{for a line plume} \tag{4a}$$

or

$$\sigma = Pr/(1+Pr) \quad \text{for a wall plume.} \tag{4b}$$

The dimensionless transverse coordinate η in equation (2) is defined [8] as

$$\eta = (y/x)[(\omega Re)^{1/2}+(\sigma Ra)^{1/5}]. \tag{5}$$

The mixed convection parameter ξ expresses the relative strength of forced convection and free convection. For pure forced convection $\xi = 0$, while for pure free convection $\xi = 1$.

With the properly defined dimensionless temperature and mixed convection parameter from scale analysis [8], we propose the following correlation equation for the centerline temperature of a line plume at any mixed convection intensity:

$$\left[\frac{1}{\theta(\xi,0)}\right]^m = \left[\frac{1-\xi}{\theta(0,0)}\right]^m \pm \left[\frac{\xi}{\theta(1,0)}\right]^m \tag{6}$$

where the plus and minus signs represent the buoyancy-assisting and -opposing flows, respectively. The centerline temperature of a pure forced convection line plume, $\theta(0,0)$, and that of a pure free convection line plume, $\theta(1,0)$, can be converted from the reported data in refs. [4, 9]. The exponent m in this correlation can be determined by comparison with available numerical data or with the previous correlation.

The correlation equation (6) can also be applied to the case of a mixed convection wall plume. The dimensionless surface temperatures $\theta(0,0)$ and $\theta(1,0)$ of the limiting cases of pure forced convection and pure free convection, respectively, are available in ref. [8].

3. CORRELATIONS OF FORCED CONVECTION AND FREE CONVECTION PLUMES

The available data of the centerline temperature of a line plume and the surface temperature of a wall plume for the pure forced convection and pure free convection cases are limited to some specific Prandtl numbers. To predict the values of $\theta(0,0)$ and $\theta(1,0)$ for any Prandtl number between 0.001 and infinity, we propose a precise correlation equation:

$$A = A_\infty\left[\frac{1+Pr}{(A_\infty/A_0)^n+Pr}\right]^{1/n} \tag{7}$$

where $A = 1/\theta(\eta=0)$ for arbitrary Prandtl number, and A_0 and A_∞ are the values of A at the limiting cases of $Pr \to 0$ and $Pr \to \infty$, respectively. The correlation is valid for both the cases of pure forced convection and pure free convection. When compared with the exact solution or the numerical data, the constant n is determined to be 6 for forced convection plumes and 5 for free convection plumes.

NOMENCLATURE

A — $1/\theta(\xi, 0)$
c_p — specific heat capacity
g — gravitational acceleration
m, n, p — exponents in equations (6), (7) and (10), respectively
Pr — Prandtl number, ν/α
Q — rate of heat released by the line source per unit length
Ra — local Rayleigh number, $g\beta T^* x^3/\alpha\nu$
Re — local Reynolds number, $u_\infty x/\nu$
T — fluid temperature
T_∞ — free-stream temperature
T^* — characteristic temperature of the line source, $Q/\rho c_p \alpha$
u_∞ — free-stream velocity
x — stream-wise coordinate measured from the line source.

Greek symbols
α — thermal diffusivity
β — thermal expansion coefficient
θ — dimensionless temperature
ν — kinematic viscosity
ξ — mixed convection parameter
ρ — density
σ — $Pr/(1+Pr)^{1/2}$ for line plume, and $Pr/(1+Pr)$ for wall plume
ω — $Pr/(1+Pr)^{1/3}$.

4. ACCURACY OF THE FORCED CONVECTION CORRELATIONS

4.1. *Forced convection line plume*

Since the present dimensionless centerline temperature $\theta(0, 0)$ is related to the dimensionless group of Haaland and Sparrow [4], $T_0(x/Pr)^{1/2}$ for $x \to 0$, by a conversion factor of $(1+Pr)^{1/6}$, their exact solution can be rewritten as

$$A = 2\pi^{1/2}(1+Pr)^{1/6} \tag{8}$$

with $A_0 = 2\pi^{1/2}$, $A_\infty = 2\pi^{1/2}(1+1000)^{1/6}$ and $n = 6$, the deviation of the correlation equation (7) from the exact solution is within 0.02% over $0.0001 \leqslant Pr \leqslant 10$, and is less than 0.17% for $Pr \leqslant 100$.

4.2. *Forced convection wall plume*

Numerical results of $\theta(0, 0)$ for $Pr = 0.001$–1000 are available in ref. [8]. The values of $\theta(0, 0)$ are 0.56429 for $Pr = 0.001$ and 0.609898 for $Pr = 1000$. With $A_0 = 1/0.56429$, $A_\infty = 1/0.609898$ and $n = 6$ in the correlation equation (7), the discrepancy between the correlation and the numerical data is less than 0.45% for $0.001 \leqslant Pr \leqslant 1000$.

5. ACCURACY OF THE FREE CONVECTION CORRELATIONS

5.1. *Free convection line plume*

The dimensionless centerline temperature of a free convection line plume is related to the dimensionless group $H(0)/Pr^{1/2}$ of refs. [4, 9] by

$$\theta(1, 0) = \left(\frac{Pr}{1+Pr}\right)^{1/10} [H(0)/Pr^{1/2}]. \tag{9}$$

The values of $H(0)/Pr^{1/2}$ have been tabulated by Haaland and Sparrow [4] for $Pr = 0.72$, 5 and ∞, and by Fujii *et al.* [9] for more Prandtl numbers from 0.01 to infinity. The converted values of $\theta(1, 0)$ are 0.44387 for $Pr = 0.1$, 0.45472 for $Pr = 0.01$, and 0.32986 for $Pr = 100$. By taking $Pr = 1000$ as infinity, the converted value of $\theta(1, 0)$ is 0.31979. With $A_0 = 1/0.455$, $A_\infty = 1/0.3198$, and $n = 5$ in the correlation equation (7), the maximum deviation between the correlation and the numerical data of ref. [9] is less than 2.8% for $0.01 \leqslant Pr \leqslant \infty$.

5.2. *Free convection wall plume*

Numerical data of $\theta(1, 0)$ for a free convection wall plume are available in ref. [8] and can be converted from the data in ref. [10]. With $A_0 = 1/0.79334$ (datum for $Pr = 0.001$), $A_\infty = 1/0.58741$ (datum for $Pr \to \infty$) and $n = 5$, the discrepancy between the correlated results and the numerical data is less than 3.5% over the range of $0.001 \leqslant Pr \leqslant \infty$.

6. ACCURACY OF THE MIXED CONVECTION CORRELATIONS

6.1. *Mixed convection line plume*

Haaland and Sparrow [4] have developed a correlation equation for the centerline temperature of a mixed convection line plume. Their correlation can be rewritten, in terms of our notation, as

$$\theta(\xi, 0) = [c/(1-\xi)(1+Pr)^{1/6}] \Big/ \left\{1 + \left[(c/d)\left(\frac{\xi}{1+\xi}\right)Pr^{-1/10}(1+Pr)^{-1/15}\right]^{1/p}\right\}^{p} \tag{10}$$

where c and p were matched [4], for $Pr = 0.72$, 5 and ∞, as $c = 0.282$; $p = 0.26 + 0.04/Pr^{1/2}$; and $d = H(0)/Pr^{1/2}$, which has been presented in refs. [4, 9] for several specific Prandtl numbers between 0.01 and infinity.

A comparison between the present correlation, equation (6) with $m = 3.5$, and the correlation of Haaland and Sparrow [4] is shown in Fig. 1 for $Pr = 0.72$, 5 and 1000. It

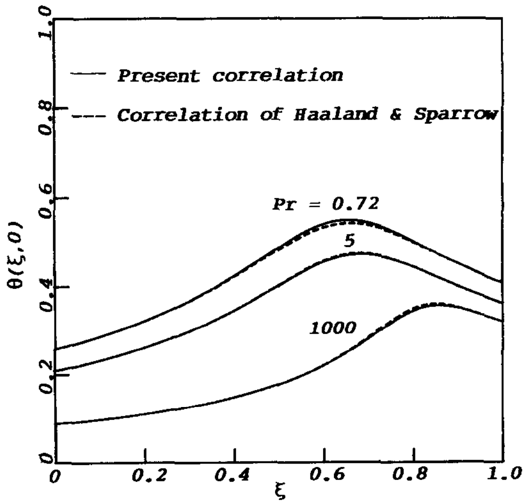

FIG. 1. Comparison of the correlations for a mixed convection wall plume.

is seen that the two correlations are in excellent agreement. The maximum deviation is less than 1.7% over the entire range of mixed convection ($0 \leqslant \xi \leqslant 1$).

6.2. *Mixed convection wall plume*

The correlation equation for the surface temperature of a mixed convection wall plume is the same as that for the centerline temperature of a mixed convection line plume. By comparing with the numerical data of $\theta(\xi, 0)$ listed in ref. [8], the exponent m and the error of the correlation equation (6) can be determined.

For the case of buoyancy-assisting flow in the entire mixed convection range ($0 \leqslant \xi \leqslant 1$), the maximum deviation of the correlation equation with $m = 3$ from the numerical results is less than 2.1% for $0.001 \leqslant Pr \leqslant 7$ and 4.1% for $7 \leqslant Pr \leqslant 1000$. The maximum error of the correlation for large Prandtl numbers ($Pr \geqslant 7$) can be reduced from 4.1% to 1.6% as $m = 3$ is replaced by $m = 3.5$.

For the case of buoyancy-opposing flow, the maximum error of the correlation is within 1.7% for $0.1 \leqslant Pr \leqslant 1000$ over the mixed convection range of $0 \leqslant \xi \leqslant 0.35$.

7. CONCLUSIONS

We have proposed comprehensive correlation equations for predicting the centerline temperature of the line plume and the surface temperature of the wall plume for the cases of laminar forced convection, free convection, and mixed convection. The correlations for the forced and the free convection plumes are very accurate over the whole range of Prandtl numbers from 0.001 to infinity. The correlations of the mixed convection plumes are of high accuracy over the entire domain of buoyancy from the forced convection limit to the free convection limit for $0.001 \leqslant Pr \leqslant \infty$. With available data of the centerline temperature or the surface temperature for the limiting cases of $Pr \to 0$ and $Pr \to \infty$, one can predict the centerline or surface temperature of the forced convection plumes and that of the free convection plumes for arbitrary Prandtl number by using equation (7). Then, the centerline or surface temperature of the mixed convection plumes can be estimated accurately from equation (6) for any flow regime of buoyancy.

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